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rgr α - CLOSED SETS IN TOPOLOGICAL SPACES

V. Jeyanthi [1] | M. Anupriya [1]

[1] Department of Mathematics, Sree Narayana Guru College, Coimbatore, India.

ABSTRACT

Here in this paper, a new class of closed set called rgrα-closed set is defined and obtained few of its characteristics.

KEY WORDS: rgrα-closed set, regular α-open set.

1. INTRODUCTION:

N.Levine[11] introduced generalized closed sets in general topology as a generalization of closed sets and a class of topological spaces called $T_{1/2}$ -spaces. Regular open and regular α-open sets have been introduced and investigated by Stone [22], A. Vadivel and K. Vairamanickam [23] respectively. Many researchers like Balachandran, Sundaram and Maki [4], Bhattacharyya and Lahiri [5], Arockiarani [2], Dunham [8], Gnanambal [9], Malghan [14], Palaniapan and Rao [19], Park [20], Arya and Gupta [3] and Devi [7], wali et al [24] have worked on generalized closed sets, their generalizations and related concepts in general topology. In this paper, we define and study the properties of regular generalized regular α-closed sets (briefly rgrα-closed). Moreover, in this paper we define rgrα-open sets and obtained some of its basic properties as results.

2. PRELIMINARIES:

Throughout this paper (X, τ) and (Y,σ) represent non empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A) and int(A) denote the closure of A and the interior of A respectively. (X, τ) will be replaced by X if there is no chance of confusion.

Let us recall the following definition which we shall require later.

Definition 2.1: A subset A of (X, τ) is called

1. a pre-open set [19] if A⊂int(cl(A)) and a pre-closed set if cl(int(A))⊂A
2. a semi-open set [12] if A⊂cl(int(A)) and a semi-closed set if int(cl(A))⊂A
3. an α-open set [7] if A⊂int(cl(int(A))) and an α-closed set if cl(int(cl(A)))⊂ A
4. a semi-preopen [1] if A⊂cl(int(cl(A))) and a semi-pre closed set if int (cl(int(A)))⊂A
5. a regular open set [19] if A=int(cl(A)) and a regular closed set if A=cl(int(A)) A subset A of (X, τ) is called clopen if it is both open and closed in (X, τ).

Definition 2.2: A subset A of a space (X, τ) is called

1. a weakly closed set (briefly, w-closed) [17] if cl(A) ⊂ U whenever A⊂U and U is semi open in X.
2. a generalized closed set (briefly, g-closed)[11] if cl(A) ⊂U whenever A⊂U and U is open in X.
3. a regular generalized closed set (briefly, rg-closed)[2] if cl(A) ⊂ U whenever A⊂U and U is regular open in X.
4. a generalized regular set [19](briefly, gr-closed) if rcl(A) ⊂ U whenever A⊂U and U is open in X.
5. a generalized pre regular closed set (briefly, gpr-closed)[9] if pcl(A) ⊂ U whenever A⊂U and U is regular open in X.
6. a regular weakly generalized closed set (briefly, rwg-closed)[17] if cl (int(A)) ⊂ U whenever A⊂U and U is regular open in X.
7. a regular generalized α- closed set (briefly, rgα-closed)[23] if αcl(A) ⊂ U whenever A⊂U and U is regular α-open in X.
8. a weakly generalized regular α-closed set (briefly, wgrα-closed)[10] if cl(int(A)) ⊂ U whenever A⊂U and U is regular α-open in X.
9. a pre-generalized regular α-closed set (briefly, pgrα-closed)[6] if pcl(A) ⊂ U whenever A⊂ U and U is regular α-open in X.

Definition 2.3: Regular closure(α-closure) of A is defined as the intersection of all regular closed sets(α-closed sets) containing A. It is denoted by rcl(A)(resp. αcl(A)).

3: rgrα -CLOSED SETS

We introduce the following definition:

Definition 3.1: A subset A of a space (X,τ) is called regular α-open set (briefly, rα-open) if there is a regular open set U such that U⊂A⊂αcl(U). The family of all regular α-open sets of (X,τ) is denoted by RαO(X).

Definition 3.2: A subset A of a space (X,τ) is called a regular generalized regular α-closed set (briefly, rgrα-closed) if rcl(A)⊂U whenever A⊂U and U is regular α-open. We denote the set of all rgrα-closed sets in (X,τ) by RGRαC(X).

Proposition 3.3: Every regular closed set is rgrα-closed.

Proof: Let A be a regular-closed set and A⊂U and U is regular α-open. Then rcl(A)=A. Therefore, rcl(A)⊂U whenever A⊂U and U is regular α-open. Hence A is rgrα-closed.

Remark 3.4: Converse of the above proposition need not be true as in the following example.

Example 3.5: Let X = {a,b,c,d}and τ = {ϕ,{a},{b},{a,b},{b,d},{a,b,d},X}. Let A= {c}.Then A is rgrα-closed but not regular closed set.

Proposition 3.6: Every gr-closed set is rgrα-closed set.

Proof: Similar to that of the proposition 3.3.

Remark 3.7: Converse of the above proposition need not be true as seen in the following example.

Example 3.8: Let X= {a,b,c,d}, τ ={ ϕ,{a},{b},{a,b},{b,d},{a,b,d}, X}. Let A ={a,b}.Then A is rgrα-closed , but not gr-closed.

Proposition 3.9: Every rgrα-closed set is rg-closed.

Proof: Let A be a rgrα-closed set in X and let A⊂U, U is regular open. Since A is rgrα-closed, rcl(A)⊂U. Since every regular closed set is closed, cl(A)⊂rcl(A). That is cl(A)⊂rcl(A)⊂U. This implies, cl(A) ⊂U whenever A⊂U & U is regular open. Hence A is rg-closed.

Remark 3.10: Converse of the above proposition need not be true as seen in the following example.

Example 3.11: Let X= {a,b,c,d}, τ ={ ϕ,{a},{b},{a,b},{b,d},{a,b,d}, X}. Let A= {c}.Then A is rg-closed, but not rgrα-closed.

Proposition 3.12: Every rgrα-closed set is gpr-closed.

Proof: Similar to that the proposition 3.9.

Remark 3.13: Converse of the above proposition need not be true as in the following example.

Example 3.14: Let $X=\{a,b,c,d\}$, $\tau=\{\phi,\{a\},\{b\},\{a,b\},\{b,d\},\{a,b,d\},X\}$. Let $A=\{c\}$.Then A is gpr-closed but not rgrα-closed.

Proposition 3.15: Every rgrα-closed set is rwg-closed.

Proof: Similar to that the proposition 3.9.

Remark 3.16: Converse of the above proposition need not be true as seen in the following example.

Example3.17: Let $X=\{a,b,c,d\}$, $\tau=\{\phi,\{a\},\{b\},\{a,b\},\{b,d\},\{a,b,d\},X\}$. Let $A=\{c\}$

Then A is rwg-closed, but not rgrα-closed.

Proposition 3.18: Every rgrα-closed set is rgα-closed.

Proof: Similar to that the proposition 3.9.

Remark 3.19: Converse of the above proposition need not be true as seen in the following example.

Example 3.20: Let $X=\{a,b,c,d\}$, $\tau=\{\phi,\{b\},\{c\},\{b,c\},\{b,c,d\},X\}$.Let $A=\{a\}$.Then A is rgα-closed but not rgrα-closed.

Proposition 3.21: Every rgrα-closed set is wgrα-closed.

Proof: Let A be a rgrα-closed set in X.Then $rcl(A)\subset U$ whenever $A\subset U$ and U is regular α-open.Since every regular closed set is closed, $cl(A)\subset rcl(A)$. Also, $int(A)\subset A$.

Then $cl(int(A))\subset cl(A)$.So $cl(int(A))\subset cl(A)\subset rcl(A)\subset U$. That is $cl(int(A))\subset U$ whenever $A\subset U$ and U is regular α-open. Hence A is wgrα-closed.

Remark 3.22: Converse of the above proposition need not be true as seen in the following example.

Example 3.23: Let $X=\{a,b,c,d\}$, $\tau=\{\phi,\{b\},\{c\},\{b,c\},\{b,c,d\},X\}$.Let $A=\{d\}$.Then A is wgrα-closed but not rgrα-closed.

Proposition 3.24: Every rgrα-closed set is pgrα-closed.

Proof: Similar to that the proposition 3.9.

Remark 3.25: Converse of the above proposition need not be true as seen in the following example.

Example 3.26: Let $X=\{a,b,c,d\}$, $\tau=\{\phi,\{b\},\{c\},\{b,c\},\{b,c,d\},X\}$.Then A is pgrα-closed, but not rgrα-closed.

Remark 3.27: The concept of closed set and rgrα-closed set are independent of one another.

Example 3.28: Let $X=\{a,b,c,d\}$, $\tau=\{\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\},\{a,b,d\},X\}$. Let $A=\{a,b\}$. Then A is rgrα-closed but not closed.

Remark 3.29: α-closed set and rgrα-closed set are independent.

Example 3.30: Let $X=\{a,b,c,d\}$, $\tau=\{\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\},\{a,b,d\},X\}$. Let $A=\{a,c\}$. Then A is rgrα-closed, but not α-closed.

Remark 3.31: w-closed set is independent of rgrα-closed set.

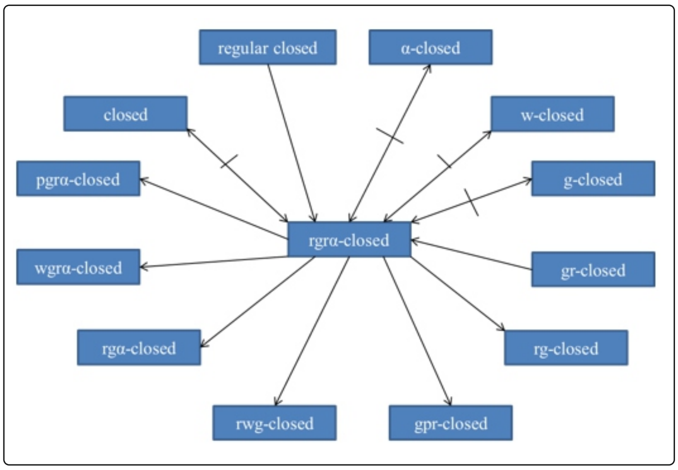
Example 3.32: Let $X=\{a,b,c,d\}$, $\tau=\{\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\},\{a,b,d\},X\}$.Let $A=\{b,d\}$. Then A is rgrα-closed, but not w-closed.

Remark 3.33: g-closed set is independent of rgrα-closed set.

Example 3.34: Let $X=\{a,b,c,d\}$, $\tau=\{\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\},\{a,b,d\},X\}$.

Let $A=\{a,b,c\}$.Then A is rgrα-closed, but not g-closed.

From the above discussions and known results we have the following implications.



Theorem 3.35: The union of two rgrα-closed sets is also rgrα-closed set.

Proof: Assume that A and B are rgrα-closed set in X. Let U be regular α-open in X such that $A\cup B\subset U$.Then $A\subset U$ and $B\subset U$.Since A and B are rgrα-closed, $rcl(A)\subset U$ and $rcl(B)\subset U$.Hence $rcl(A\cup B)=rcl(A)\cup rcl(B)\subset U$.That is $rcl(A\cup B)\subset U$. Therefore, $A\cup B$ is also rgrα-closed set in X.

Remark 3.36: The intersection of two rgrα-closed sets in X is generally not rgrα-closed set in X.

Example 3.37: Let $X=\{a,b,c,d,e\}$ with topology $\tau=\{\phi,\{a\},\{b\},\{a,b\},\{b,c,d\},\{a,b,c,d\},X\}$.If $A=\{a,b.e\}$ and $B=\{a,d,e\}$. Then A and B are rgrα-closed sets in X, but $A\cap B=\{a,e\}$ is not a rgrα-closed set in X.

Theorem 3.38 : If a subset A of X is rgrα-closed set in X. Then rcl(A)-A does not contain any non empty regular α-open set in X.

Proof: Suppose that A is rgrα-closed set in X.We prove the result by contradiction .Let U be a regular α-closed set and $U\subset rcl(A)-A$ and $U\neq\phi$.

Now $U\subset rcl(A)-A$.Therefore, $U\subset X-A$.This implies, $A\subset X-U$.Since U is regular α-closed set, X-U is regular α-open in X.Since A is rgrα-closed set in X, by definition we have $rcl(A)\subset X-U$. So $U\subset X-rcl(A)$.Also $U\subset rcl(A)$.

Therefore, $U\subset rcl(A)\cap(X-rcl(A))=\phi$. This shows that $U=\phi$ which is a contradiction.Hence rcl(A)-A does not contain any non-empty regular α-open sets in X.

Corollary 3.39: If a subset A of X is rgrα-closed in X, then rcl(A)-A does not contain any non empty regular closed set in X, but not conversely.

Proof: Let F be a regular closed set such that $F\subset rcl(A)-A$.Then $F\subset X-A$.Therefore, $A\subset X-F$,X-F is regular open and hence regular α-open. Since A is rgrα-closed and X-F is regular α-open, then $rcl(A)\subset X-F$.That is,$F\subset X-rcl(A)$.Hence $F\subset rcl(A)\cap(X-rcl(A))=\phi$.That is $F=\phi$.

Remark 3.40: If rcl(A)-A contains no non empty regular closed set in X, then A need not be rgrα-closed set in X.

Example3.41: Let $X=\{a,b,c,d\}$, $\tau=\{\phi,\{a\},\{c,d\},\{a,c,d\}\ X\}$. Let $A=\{c,d\}$.Then $rcl(A)=\{b,c,d\}$. $rcl(A)-A=\{b\}$ which does not contain any non empty regular closed set. Also, A is not rgrα-closed.

Theorem 3.42: For an element $x\epsilon X$, the set X-{x} is rgrα-closed or regular α-open.

Proof: Suppose X-{x} is not a regular α-open set.Then X is the only regular α-open set containing X-{x}.This implies, $rcl(X-\{x\})\subset X$. Hence X-{x} is rgrα-closed set.

Theorem 3.43: If A is regular open and rgrα-closed then A is regular closed and hence clopen.

Proof: Suppose A is regular open and rgrα-closed.Since every regular open set is regular α-open and $A\subset A$,we have $rcl(A)\subset A$.Also $A\subset rcl(A)$.Therefore

rcl(A)=A. Since A is both regular open and regular closed, hence A is clopen.

Theorem 3.44: If A is rgrα-closed subset of X such that A⊂B⊂rcl(A), then B is rgrα-closed set in X.

Proof: Let A is rgrα-closed subset of X such that A⊂B⊂rcl(A). Let U be a regular α-open set of X such that B⊂U. Then A ⊂U. Since A is rgrα-closed, we have rcl(A) ⊂U. Now rcl(B) ⊂rcl(rcl(A))=rcl(A) ⊂U. That is rcl(B) ⊂U. Therefore, B is rgrα-closed set in X.

Theorem 3.45: Let A be a rgrα-closed set in X then A is regular closed if and only if rcl(A)-A is regular α-open.

Proof: Suppose A is a regular closed in X. Then rcl(A)=A. So rcl(A)-A= ϕ, which is regular α-open set in X. Conversely, Suppose rcl(A)-A is a regular α-open set in X. Since A is rgrα-closed, by theorem 3.38, rcl(A)-A does not contain any non empty regular α-open in X. Then rcl(A)-A= ϕ. That is rcl(A)=A. Hence A is regular closed set in X.

Theorem 3.46: If A is regular open and regular closed (that is clopen) in X , then A is rgrα-closed set in X.

Proof: Let A be a regular open and regular closed set in X, that is A is clopen. Since every regular closed set is rgrα-closed and every regular open set is regular α-open, rcl(A)=A⊂U whenever A⊂U and U is regular α-open. Hence A is rgrα-closed set in X.

Theorem 3.47: If a subset A of topological space X is both regular α-open and rgrα-closed, then it is regular closed.

Proof: Suppose a subset A of a is a subset of topological space X is both regular α-open and rgrα-closed. Now A⊂A and we know that A⊂ rcl(A). Then rcl(A) ⊂A. Hence A is regular closed.

Corollary 3.48: Let A be regular α-open and rgrα-closed subset in X. Suppose that F is regular closed set in X. Then A∩F is an rgrα-closed set in X.

Proof: Let A be a regular α-open set and rgrα-closed subset in X and F be regular closed. By theorem 3.47, A is regular closed. So, A∩F is regular closed. Since every regular closed set is rgrα- closed, A∩F is also rgrα- closed. Hence A∩F is rgrα-closed set in X.

Theorem 3.49: If A is both regular α-open and rgrα-closed in X, then A is g-closed.

Proof: Let A be a regular α-open and rgrα-closed set. Then rcl(A) ⊂A. Also A⊂rcl(A). Hence rcl(A)=A. This implies, A is regular closed and then closed. Hence A is g-closed.

Theorem 3.50: In a topological space X, RαO(X)={X, ϕ}, then every subset of X is rgrα-closed set.

Proof: Let X be a topological space and RαO(X)={X, ϕ}. Let A be any subset of X. Suppose A= ϕ. Then ϕ is rgrα-closed set in X. Suppose A≠ ϕ. Then X is the only regular α-open set containing A. So, rcl(A) ⊂X. Hence A is rgrα-closed set in X.

Remark 3.51: The converse of the theorem need not be true.

Example 3.52: Let X={a,b,c,d}, τ ={ ϕ,{a,b},{c,d}, X}. Then every subset of (X, τ) is rgrα-closed set in X. But Rαo (X, τ) ={ ϕ,{a,b},{c,d}, X}.

4. rgrα-OPEN SETS:

In this section, we introduce and study rgrα-open sets in topological spaces and obtain some of their properties.

Definition 4.1: A subset A of (X,τ) is called regular generalized regular α-open(briefly, rgrα-open) in X if A^c is rgrα-closed in X. We denote the family of all rgrα-open sets in (X, τ) by RGRαO(X).

Proposition 4.2: Every regular open set is rgrα-open set, but not conversely.

Proof: Let A be a regular open set in X. Then A^c is regular closed set. By proposition 3.3, A^c is rgrα-closed set. Therefore A is rgrα-open set in X.

Example 4.3: Let X={a,b,c,d}, τ={ ϕ,{c},{a,c},{c,d},{a,c,d},{b,c,d},X}. Let A ={d}. Then A is rgrα-open but not regular open.

Proposition 4.4: If a subset A of a space X is gr-open then it is rgrα-open, but not conversely.

Proof: Let A be a gr-open set in space X. Then A^c is gr-closed set. By proposition 3.6, A^c is rgrα-closed set as every gr-closed set is rgrα-closed. Therefore A is rgrα-open.

Example 4.5: Let X={a,b,c,d}, τ={ ϕ,{c},{a,c},{c,d},{a,c,d},{b,c,d},X}. Let A = {c}. Then A is rgrα-open, but not gr-open.

Proposition 4.6: If a subset A of a space X is rgrα-open, then it is rg-open set in X, but not conversely.

Proof: Let A be rgrα-open set in space X. Then A^c is rgrα-closed set in X. By proposition 3.9, A^c is rg-closed set in X. Therefore , A is rg-open set in X.

Example 4.7: Let X={a,b,c,d}, τ={ ϕ,{c},{d},{a,d},{c,d},{a,c,d},{b,c,d},X}. Let A={a,b,d}. Then rg-open but not rgrα-open.

Proposition 4.8: If a subset A of a space X is rgrα-open, then it is gpr-open set in X, but not conversely.

Proof: Let A be rgrα-open set in X. Then A^c is rgrα-closed set in X. By proposition 3.12 , A^c is gpr-closed set in X. A is gpr-open set in X.

Example 4.9: Let X={a,b,c,d}, τ={ ϕ,{c},{d},{a,d},{c,d},{a,c,d},{b,c,d},X}. Let A={a,b,d}. Then A is gpr -open, but not rgrα-open.

Proposition 4.10: If a subset A of a space X is rgrα-open, then it is rwg-open set in X, but not conversely.

Proof: Let A be rgrα-open set in X. Then A^c is rgrα-closed set in X. By proposition 2.15 , A^c is rwg-closed set in X. A is rwg-open set in X.

Example 4.11: Let X={a,b,c,d}, τ={ ϕ,{c},{d},{a,d},{c,d},{a,c,d},{b,c,d},X}. Let A={a,b,d}. Then A is rwg-open, but not rgrα-open.

Proposition 4.12: If a subset A of a space X is rgrα-open, then it is rgα-open set in X, but not conversely.

Proof: Let A be rgrα-open set in X. Then A^c is rgrα-closed set in X. By proposition 3.18, A^c is rgα-closed set in X. Therefore A is rgα-open set in X.

Example 4.13: Let X={a,b,c,d}, τ={ ϕ,{a},{a,d},{a,b,d},{a,c,d},X}. Let A={a,b,c}. Then A is rgα -open, but not rgrα-open.

Proposition 4.14: If a subset A of a space X is rgrα-open, then it is wgrα-open set in X, but not conversely.

Proof: Let A be rgrα-open set in X. Then A^c is rgrα-closed set in X. By proposition 3.21, A^c is wgrα-closed set in X. Therefore A is wgrα-open set in X.

Example 4.15: Let X={a,b,c,d}, τ={ ϕ,{a},{a,d},{a,b,d},{a,c,d},X}. Let A={b,c,d}. Then A is wgrα -open, but not rgrα-open.

Proposition 4.16: If a subset A of a space X is rgrα-open, then it is pgrα-open set in X, but not conversely.

Proof: Let A be rgrα-open set in X. Then A^c is rgrα-closed set in X. By proposition 3.24, A^c is pgrα-closed set in X. Therefore A is pgrα-open set in X.

Example 4.17: Let X={a,b,c,d}, τ={ ϕ,{a},{a,d},{a,b,d},{a,c,d},X}. Let A={a,b,c}. Then A is pgrα -open, but not rgrα-open.

Theorem 4.18: If a subset A and B are rgrα-open sets in a space X, then A∩B is also rgrα-open set in X.

Proof: If A and B are rgrα-open sets in space X. Then A^c and B^c are rgrα-closed sets in a space X. By theorem 3.35, $A^c \cup B^c$ is also rgrα-closed set in X. $(A\cap B)^C = A^c \cup B^c$ ie; $(A\cap B)^C$ is a rgrα-closed set in X. Therefore, A∩B is rgrα-open set in X.

Remark 4.19: The union of two rgrα-open sets in X is generally not an rgrα-open set in X.

Example 4.20: Let X={a,b,c,d,e} and τ ={ϕ,{e},{a.e},{c,d,e},{a,c,d,e},{b,c,d,e}, X}.

If A = {c,d} and B={b,c}. Then A and B are rgrα-open sets in X, but AUB = {b,c,d} is not a rgrα-open set in X.

Theorem 4.21: If a set A is rgrα-open in a space X, then G=X, whenever G is

regular α-open and int(A) ∪ A^c ⊂ G.

Proof: Suppose that A is rgrα-open in X. Let G be regular α-open and int(A) U A^c ⊂ G.This implies G^c ⊂ (int(A) U $A^c)^c$ = $(int(A))^c$ ∩ A ie; G^c ⊂ $(int(A))^c$ - A^c. Since $(int(A))^c$ = cl(A^c).Thus G^c ⊂ cl(A^c)-A^c.Now G^c is also becomes regular α-open set, since A^c is rgrα-closed. We know that A ⊂ cl(A). $[cl (A)]^c$ ⊂ A^c, $[cl (A)]^c$ - A^c ⊂ ϕ. G^c = ϕ and hence G=X.

Theorem 4.22: Every singleton point set in a space is either rgrα-open or rα-open.

Proof: Let X be a topological space.Let x∈X. To prove {x} is either rgrα-open or rα-open, it is enough to prove X-{x} is either rgrα-closed or rα-open. Suppose X-{x} is not a regular α-open set.Then X is the only regular α-open set containing X-{x}.This implies, rcl(X-{x})⊂X. Hence X-{x} is rgrα-closed set. Therefore, {x} is rgrα-open.

Theorem 4.23: A⊂X is rgrα-open if F⊂ rint(A), whenever F is regular α-closed and F⊂A.

Proof: Let A be rgrα-open.Let F be regular closed and F⊂A.Then X-A⊂X-F, where X-F is regular α-open.Since A is rgrα-open implies rcl(X-A) ⊂X-F. This implies X-rint(A)⊂X-F.Hence F⊂rint A.Conversely:Suppose F is regular α-closed & F⊂A. Let X-A⊂U,where U is regular α-open.Then X-A⊂U & X- U is regular α-closed. By hypothesis, X-U ⊂rint(A).

ie, X-rint(A)⊂U.This implies rcl(X-A) ⊂U. Then X-A is rgrα-closed. Hence A is rgrα-open.

Theorem 4.24: If rint(A)⊂B⊂A & A is rgr-open then B is rgrα-open.

Proof: rint(A)⊂B⊂A implies (X-A)⊂(X-B) ⊂ (X-rint(A)).ie; (X-A) ⊂(X-B) ⊂rcl(X-A).Since (X-A) is rgrα-closed, by theorem 3.44, (X-B) is rgrα-closed. So B is rgrα-open.

Corollary 4.25: For any A⊂X,rint(rcl(A)-A)= ϕ

Theorem 4.26: If A⊂X is rgrα-closed then rcl(A)-A is rgrα-open.

Proof: Let A be rgrα-closed.Let F be a regular α-closed set and F⊂rcl(A)-A.

By theorem 3.38, F= ϕ.So F⊂rint(rcl(A)-A).This shows rcl(A)-A is rgrα-open.

Definition 4.27: A space (X, τ) is called rgrα-$T_{1/2}$ space if every rgrα-closed set is regular closed.

Theorem 4.28: For a space (X, τ) the following are equivalent.

(1) X is rgrα-$T_{1/2}$ space.

(2) Every singleton is either regular closed or regular open.

Proof: Suppose{x} is not a regular α-closed subset for some x∈X.Then X-{x} is not regular α-open. Hence X is the only regular α-open set containing X-{x}.This implies rcl(X-{x})⊂X-{x}. Therefore X-{x} is rgrα-closed. Since (X, τ) is rgrα-$T_{1/2}$, X-{x} is regular closed. Then{x} is regular open. Hence (1) implies (2).Let A be a rgrα-closed subset of (X, τ) and x∈rcl(A).We will show that x∈A.

If {x} is regular α-closed and x≠A, then {x}∈(rcl(A)-A). Thus (rcl(A)-A) contains a non empty regular α-closed, which is a contradiction to theorem 3.38. So x∈A.

If {x} is regular open since x∈rcl(A) then for every regular-open set U contains x. We have U∩A= ϕ. But{x} is regular-open then{x}∩A≠ ϕ. Hence x∈A.

So in both cases, we have x∈A. Therefore A is regular-closed set.Hence (2) implies(1).

References

1. D.Andrijevi, Semi-preopen sets, Mat. Vesnil, 38(1986), 24-32.
2. I.Arockiarani, Studies on generalizations of generalized closed sets and maps in topological spaces, Ph.D Thesis, Bharathiar University,Coimbatore,(1997).
3. S.P.Arya and R.Gupta, On strongly continuous mappings, Kyungpook Math., J.14(1974),131-143.
4. K.Balachandran, P.Sundaram and H.Maki, On generalized continuous maps in topological spaces, Mem. I ac Sci. Kochi Univ. Math., 12(1991),5-13.
5. P.Bhattacharyya and B.K. Lahiri, Semi-generalized closed sets in topology, Indian J.Math., 29(1987), 376-382.
6. K.Binoy Balan and C.Janaki on pgrα-closed sets in topological spaces/ IJMA-3(5),May(2012).
7. R.Devi, K.Balachandran and H.Maki, On generalized α-continuous maps, Far. East J.Math. Sci. Special Volume, part 1(1997), 1-15
8. W.Dunham, A new closure operator for non-T1 topologies, Kyungpook Math. J., 22(1982), 55-60.
9. Y.Gnanambal, On generalized pre-regular closed sets in topological spaces, Indian J. Pure Appl. Math., 28(1997), 351-360.
10. A.Jayalakshmi and C.Janaki(IJERA) Vol.3, Issue 2, March- April(2013).
11. N.Levine, Generalized closed sets in topology, Rend. Circ. Mat. Palermo, 19(1970), 89-96.
12. N.Levine, Semi-open sets and semi-continuity in topological spaces, Amer.Math.Monthly 70(1963), 36-41.
13. H.Maki, R.Devi and K.Balachandran, Associated topologies of generalized α-closed sets and α-generalized closed set, mem.fac.sci.kochi univ.ser.A math,15(1994),51-63.
14. S.R.Malghan, Generalized closed maps, J.Karnatak Univ. Sci., 27(1982), 82-88.
15. A.S.Mashhour, M.E. Abd. EI-Monsef and S.N.EI-Deeb, On pre-continuous mappings an weak pre-continuous mappings, Proc. Math. Phy.Soc. Egypt., 53(1982), 47-53.
16. O.Njastad, On some classes of nearly open sets, Pacific J.Math., 15(1965), 961-970.
17. N.Nagaveni, Studies on generalization of homomorphisms in topological spaces, Ph.D. thesis, Bharathiar University, Coimbatore(1999).
18. S.R.Malghan, Generalized closed maps, J.Karnatak Univ. Sci., 27(1982), 82-88.
19. N.Palaniappan and K.C.Rao, Regular generalized closed sets, Kyungpook, Math. J., 33(1993), 211-219.
20. J.K. Park and J.H. Park, Mildly generalized closed sets, almost normal and mildly normal spaces, Chaos, Solutions and Fractals 20(2004), 1103-1111.
21. M.Sheik John, On w-closed sets in topology, Acta Ciencia India, 4(2000), 389-392.
22. Stone.M, Application of the theory of Boolean Rings to General Topology, Trans. Amer. Math.Soc., 41(1973), 374-481.
23. A. Vadivel and K.Vairamanickam, rgα-Closed Sets and rgα-Open Sets in Topological spaces, Int. Journal of Math. Analysis, Vol. 3, 2009, No. 37, 1803-1819.
24. R.S.Wali, Some topics in general and fuzzy topological spaces, Ph.D., Thesis, Karnatak University, Karnataka(2006).